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Central Charges in Non(anti)commutative $\mathcal{N} = 2$ Supersymmetric $U(N)$ Gauge Theory

Katsushi Ito and Hiroaki Nakajima

*Department of Physics
Tokyo Institute of Technology
Tokyo, 152-8551, Japan*

Abstract

We study the central charge of the deformed $\mathcal{N} = (1, 0)$ supersymmetry algebra in non(anti)commutative $\mathcal{N} = 2$ supersymmetric $U(N)$ gauge theory. In the cases of $\mathcal{N} = 1/2$ superspace and $\mathcal{N} = 2$ harmonic superspace with the singlet deformation, we find that the central charge is deformed by the non(anti)commutative parameters but depends on the electric and magnetic charges. For generic deformation of $\mathcal{N} = 2$ harmonic superspace, we compute the $O(C)$ correction to the central charges in the case of $U(1)$ gauge group.

Supersymmetric field theories in non(anti)commutative superspace [1, 2] arise as the low-energy effective field theories on the D-branes in the graviphoton background [3, 4, 5] and have been extensively studied in the last few years. These theories are defined in Euclidean superspace by using the \ast -product for the supercoordinates. In particular supersymmetric field theories in non(anti)commutative $\mathcal{N} = 1$ superspace ($\mathcal{N} = 1/2$ superspace) have been investigated both perturbatively and nonperturbatively [6, 7, 8, 9].

In the case of $\mathcal{N} = 2$ superspace, there exists a variety of deformations [10, 11, 12, 13]. For generic Q -deformation, $\mathcal{N} = (1, 1)$ supersymmetry is deformed to $\mathcal{N} = (1, 0)$ [12, 13]. But for some particular deformation parameters, in which only $\mathcal{N} = 1$ subspace is deformed, it is shown that the $\mathcal{N} = (1, 0)$ supersymmetry enhances to $\mathcal{N} = (1, 1/2)$ [12]. The deformed $\mathcal{N} = (1, 1/2)$ supersymmetry has been constructed explicitly in [14] for the $U(1)$ gauge theory.

For $\mathcal{N} = 2$ supersymmetric $U(N)$ gauge theory in non(anti)commutative $\mathcal{N} = 1$ superspace, the deformed $\mathcal{N} = (1, 1/2)$ supersymmetry is constructed in [15]. It is interesting to study the role of the deformed supersymmetry at the quantum level. Since non(anti)commutative field theories do not have Poincaré invariance, the supersymmetry algebra could get nontrivial corrections, which avoid the Haag-Łopuszański-Sohnius no-go theorem [16]. In $\mathcal{N} = 1/2$ supersymmetric field theories, the central charge of the deformed supersymmetry algebra was studied in [17, 18]. In particular, for $\mathcal{N} = 1/2$ Wess-Zumino (WZ) model, it was shown that the formula of the central charge associated with the domain wall is not deformed but the non(anti)commutative effects enter through the deformed equations of motion[18].

In $\mathcal{N} = 2$ supersymmetric gauge theory, Witten and Olive [19] have shown that there exists a central extension in $\mathcal{N} = 2$ supersymmetry algebra and its central charge is related to the electric and magnetic charges of the monopoles or dyons of the theory (See [20] for reviews). Their BPS property is very important to study the strong coupling physics of the theory [21].

In this paper we will study the effects of non(anti)commutativity on the deformed $\mathcal{N} = (1, 0)$ supersymmetry algebra of $\mathcal{N} = 2$ supersymmetric $U(N)$ gauge theory in $\mathcal{N} = 1/2$ superspace. We find that new central charges appear in the algebra but they depend on the electric and magnetic charges and the vacuum expectation value of the

Higgs fields. We then extend this result to the non(anti)commutative $\mathcal{N} = 2$ harmonic superspace. Since the exact $U(N)$ action is only known for the singlet deformation [22], we calculate the $\mathcal{N} = (1, 0)$ algebra based on this theory. For generic deformation, the $O(C)$ $U(1)$ action has been obtained in [13]. We will calculate the $O(C)$ correction to the central charge.

We begin with reviewing $\mathcal{N} = 2$ supersymmetric $U(N)$ gauge theory in non(anti)commutative $\mathcal{N} = 1$ superspace. Let $(x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$ ($\mu = 0, \dots, 3$, $\alpha, \dot{\alpha} = 1, 2$) be supercoordinates of $\mathcal{N} = 1$ Euclidean superspace and $\sigma_{\alpha\dot{\alpha}}^\mu$ and $\bar{\sigma}^{\mu\dot{\alpha}\alpha}$ Dirac matrices [23]. We note that in Euclidean spacetime chiral and antichiral fermions transform independently under the Lorentz transformations. $Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}} \partial_\mu$ and $\bar{Q}^{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i\theta_\alpha \bar{\sigma}^{\mu\dot{\alpha}\alpha} \partial_\mu$ are supercharges. $D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}} \partial_\mu$ and $\bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta_\alpha \bar{\sigma}^{\mu\dot{\alpha}\alpha} \partial_\mu$ are the supercovariant derivatives. $\sigma^{\mu\nu} = \frac{1}{4}(\sigma^\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}^\mu)$ and $\bar{\sigma}^{\mu\nu} = \frac{1}{4}(\bar{\sigma}^\mu \sigma^\nu - \bar{\sigma}^\nu \sigma^\mu)$ are the Lorentz generators.

The non(anti)commutativity in $\mathcal{N} = 1$ superspace is introduced by the $*$ -product:

$$f * g(x, \theta, \bar{\theta}) = f(x, \theta, \bar{\theta}) \exp\left(-\frac{1}{2} \overleftarrow{Q}_\alpha C^{\alpha\beta} \overrightarrow{Q}_\beta\right) g(x, \theta, \bar{\theta}). \quad (1)$$

Using this $*$ -product, the anticommutation relations for θ become

$$\{\theta^\alpha, \theta^\beta\}_* = C^{\alpha\beta} \quad (2)$$

while the chiral coordinates $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$ and $\bar{\theta}$ are still commuting and anticommuting coordinates, respectively.

$\mathcal{N} = 2$ supersymmetric $U(N)$ gauge theory in this deformed superspace was formulated in [6]. It can be constructed by vector superfields V , chiral superfields Φ and an anti-chiral superfields $\bar{\Phi}$, where Φ and $\bar{\Phi}$ belong to the adjoint representation of $U(N)$. We introduce the basis t^a ($a = 1, \dots, N^2$) of the Lie algebra of $U(N)$, normalized as $\text{tr}(t^a t^b) = k\delta^{ab}$. The Lagrangian is

$$\mathcal{L} = \frac{1}{k} \int d^2\theta d^2\bar{\theta} \text{tr}(\bar{\Phi} * e^V * \Phi * e^{-V}) + \frac{1}{16kg^2} \text{tr} \left(\int d^2\theta W^\alpha * W_\alpha + \int d^2\bar{\theta} \bar{W}_{\dot{\alpha}} * \bar{W}^{\dot{\alpha}} \right), \quad (3)$$

where g denotes the coupling constant. $W_\alpha = -\frac{1}{4}\bar{D}^2 e^{-V} D_\alpha e^V$ and $\bar{W}_{\dot{\alpha}} = \frac{1}{4}D^2 e^{-V} \bar{D}_{\dot{\alpha}} e^V$ are the chiral and antichiral field strengths. Note that multiplication of superfields are defined by the $*$ -product. This Lagrangian is invariant under the gauge transformations $\Phi \rightarrow e^{-i\Lambda} * \Phi * e^{i\Lambda}$, $\bar{\Phi} \rightarrow e^{-i\bar{\Lambda}} * \bar{\Phi} * e^{i\bar{\Lambda}}$ and $e^V \rightarrow e^{-i\bar{\Lambda}} * e^V * e^{i\Lambda}$. To write down the

Lagrangian in terms of component fields, it is convenient to take the WZ gauge as in the commutative case. Since the $*$ -product deforms the gauge transformation, it is necessary to redefine the component fields such that these transform canonically under the gauge transformation[2, 6]. For $\mathcal{N} = 2$ $U(N)$ theory, these superfields in the WZ gauge are

$$\begin{aligned}
\Phi(y, \theta) &= A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y), \\
\bar{\Phi}(\bar{y}, \bar{\theta}) &= \bar{A}(\bar{y}) + \sqrt{2}\bar{\theta}\bar{\psi}(\bar{y}) + \bar{\theta}\bar{\theta} \left(\bar{F} + iC^{\mu\nu}\partial_\mu \{v_\nu, \bar{A}\} - \frac{1}{4}C^{\mu\nu} [v_\mu, \{v_\nu, \bar{A}\}] \right) (\bar{y}), \\
V(y, \theta, \bar{\theta}) &= -\theta\sigma^\mu\bar{\theta}v_\mu(y) + i\theta\theta\bar{\theta}\bar{\lambda}(y) - i\bar{\theta}\bar{\theta}\theta^\alpha \left(\lambda_\alpha + \frac{1}{4}\varepsilon_{\alpha\beta}C^{\beta\gamma} \{(\sigma^\mu\bar{\lambda})_\gamma, v_\mu\} \right) (y) \\
&\quad + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}(D - i\partial^\mu v_\mu)(y).
\end{aligned} \tag{4}$$

Here $\bar{y}^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}$ are the antichiral coordinates and $C^{\mu\nu} = C^{\alpha\beta}\varepsilon_{\beta\gamma}(\sigma^{\mu\nu})_\alpha{}^\gamma$. Since $\sigma^{\mu\nu}$ is self-dual, $C^{\mu\nu}$ is also self-dual. Substituting (4) into the Lagrangian (3), we obtain the deformed Lagrangian written in terms of component fields. In this expression, however, normalizations of two fermions ψ and λ are different. In order to see symmetries between two fermions manifestly, it is useful to rescale V to $2gV$ and $C^{\alpha\beta}$ to $\frac{1}{2g}C^{\alpha\beta}$. Then the Lagrangian takes the form

$$\begin{aligned}
\mathcal{L} &= \frac{1}{k}\text{tr} \left(-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}F^{\mu\nu}\tilde{F}_{\mu\nu} - i\bar{\lambda}\bar{\sigma}^\mu D_\mu\lambda + \frac{1}{2}\tilde{D}^2 \right. \\
&\quad \left. - (D^\mu\bar{A})D_\mu A - i\bar{\psi}\bar{\sigma}^\mu D_\mu\psi + \bar{F}F - i\sqrt{2}g[\bar{A}, \psi]\lambda - i\sqrt{2}g[A, \bar{\psi}]\bar{\lambda} - \frac{g^2}{2}[A, \bar{A}]^2 \right) \\
&\quad + \frac{1}{k}\text{tr} \left(-\frac{i}{2}C^{\mu\nu}F_{\mu\nu}\bar{\lambda}\bar{\lambda} + \frac{1}{8}|C|^2(\bar{\lambda}\bar{\lambda})^2 \right. \\
&\quad \left. + \frac{i}{2}C^{\mu\nu}F_{\mu\nu}\{\bar{A}, F\} - \frac{\sqrt{2}}{2}C^{\alpha\beta}\{D_\mu\bar{A}, (\sigma^\mu\bar{\lambda})_\alpha\}\psi_\beta - \frac{1}{16}|C|^2[\bar{A}, \bar{\lambda}][\bar{\lambda}, F] \right), \tag{5}
\end{aligned}$$

where $F_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu + ig[v_\mu, v_\nu]$, $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$, $|C|^2 = C^{\mu\nu}C_{\mu\nu}$ and $D_\mu\lambda = \partial_\mu\lambda + ig[v_\mu, \lambda]$ etc. We have also introduced an auxiliary field \tilde{D} defined by $\tilde{D} = D + g[A, \bar{A}]$ in order to see undeformed $\mathcal{N} = 2$ supersymmetry in a symmetric way. It is shown in [6, 15] that the action is invariant under the deformed $\mathcal{N} = (1, 1/2)$ supersymmetry

$$\begin{aligned}
\delta_\xi v_\mu &= i\xi\sigma_\mu\bar{\lambda}, \\
\delta_\xi\lambda_\alpha &= i\xi_\alpha\tilde{D} - ig\xi_\alpha[A, \bar{A}] + (\sigma^{\mu\nu}\xi)_\alpha \left(F_{\mu\nu} + \frac{i}{2}C_{\mu\nu}\bar{\lambda}\bar{\lambda} \right), \quad \delta_\xi\bar{\lambda} = 0, \\
\delta_\xi\tilde{D} &= -\xi\sigma^\mu D_\mu\bar{\lambda} + \sqrt{2}g[\xi\psi, \bar{A}],
\end{aligned}$$

$$\begin{aligned}
\delta_\xi A &= \sqrt{2}\xi\psi, & \delta_\xi\psi &= \sqrt{2}\xi F, & \delta_\xi F &= 0, \\
\delta_\xi \bar{A} &= 0, \\
\delta_\xi \bar{\psi} &= \sqrt{2}i\bar{\sigma}^\mu\xi D_\mu\bar{A}, \\
\delta_\xi \bar{F} &= i\sqrt{2}\xi\sigma^\mu D_\mu\bar{\psi} - 2gi[\bar{A}, \xi\lambda] + C^{\mu\nu}D_\mu\{\bar{A}, \xi\sigma_\nu\bar{\lambda}\},
\end{aligned} \tag{6}$$

$$\begin{aligned}
\delta_\eta v_\mu &= -i\eta\sigma_\mu\bar{\psi} - \frac{\sqrt{2}}{2}C^{\alpha\beta}\eta_\alpha\{\bar{A}, (\sigma_\mu\bar{\lambda})_\beta\}, \\
\delta_\eta\lambda^\alpha &= \sqrt{2}\eta^\alpha\bar{F} \\
&\quad - \frac{\sqrt{2}}{2}C^{\alpha\beta}\eta_\beta\{\tilde{D}, \bar{A}\} - \frac{\sqrt{2}i}{2}C^{\alpha\beta}(\sigma^{\mu\nu}\eta)_\beta\{F_{\mu\nu}, \bar{A}\} - \frac{\sqrt{2}g}{2}C^{\alpha\beta}\eta_\beta\{\bar{A}, [\bar{A}, A]\} \\
&\quad + \frac{\sqrt{2}}{4}\det C\left(\{\bar{\lambda}\bar{\lambda}, \bar{A}\} + 2\bar{\lambda}_{\dot{\alpha}}\bar{A}\bar{\lambda}^{\dot{\alpha}}\right)\eta^\alpha, \\
\delta_\eta\bar{\lambda} &= \sqrt{2}i\bar{\sigma}^\mu\eta D_\mu\bar{A}, \\
\delta_\eta\tilde{D} &= -\eta\sigma^\mu D_\mu\bar{\psi} - \sqrt{2}g[\eta\lambda, \bar{A}] - \frac{\sqrt{2}}{2}iC^{\alpha\beta}\eta_\beta D_\mu\{\bar{A}, (\sigma^\mu\bar{\lambda})_\alpha\} - igC^{\alpha\beta}\eta_\beta\{\bar{A}, [\bar{A}, \psi_\alpha]\}, \\
\delta_\eta A &= \sqrt{2}\eta\lambda + iC^{\alpha\beta}\eta_\beta\{\psi_\alpha, \bar{A}\}, \\
\delta_\eta\psi^\alpha &= i\eta^\alpha\tilde{D} + ig\eta^\alpha[A, \bar{A}] - \varepsilon^{\alpha\beta}(\sigma^{\mu\nu}\eta)_\beta F_{\mu\nu} - iC^{\alpha\beta}\eta_\beta\{(\bar{\lambda}\bar{\lambda}) - \{\bar{A}, F\}\}, \\
\delta_\eta F &= i\sqrt{2}\eta\sigma^\mu D_\mu\bar{\lambda} + 2gi[\bar{A}, \eta\psi], \\
\delta_\eta\bar{A} &= 0, \\
\delta_\eta\bar{\psi}_{\dot{\alpha}} &= C^{\alpha\beta}\eta_\beta\sigma_{\alpha\dot{\alpha}}^\mu\{\bar{A}, D_\mu\bar{A}\}, \\
\delta_\eta\bar{F} &= \sqrt{2}gC^{\alpha\beta}\eta_\beta\{\bar{A}, [\bar{A}, \lambda_\alpha]\} + \frac{\sqrt{2}i}{4}\det C\left[3\{\bar{A}, \{\eta\sigma^\mu\bar{\lambda}, D_\mu\bar{A}\}\} \right. \\
&\quad \left. + 2D_\mu\bar{A}\bar{A}\eta\sigma^\mu\bar{\lambda} + 2\eta\sigma^\mu\bar{\lambda}\bar{A}D_\mu\bar{A} + 2\{\bar{A}, \{\eta\sigma^\mu D_\mu\bar{\lambda}, \bar{A}\}\}\right],
\end{aligned} \tag{7}$$

where we have written down only the part of $\mathcal{N} = (1, 0)$ supersymmetry.

We now compute the Noether currents associated with deformed $\mathcal{N} = (1, 0)$ supersymmetry transformations δ_ξ and δ_η . Let X_ξ^μ be the total derivative term obtained from the variation of the Lagrangian associated with the transformation δ_ξ :

$$\delta_\xi\mathcal{L} = \partial_\mu X_\xi^\mu.$$

Then the supercurrent $N_{1\alpha}^\mu$ is defined by

$$\xi^\alpha N_{1\alpha}^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi_A)}\delta_\xi\varphi_A - X_\xi^\mu \tag{8}$$

where φ_A are component fields in the WZ gauge. The other supercurrent $N_{2\alpha}^\mu$ associated with the transformation δ_η is defined in a similar way. From the Lagrangian (5) and the transformations (6), we get

$$\begin{aligned}\xi N_1^\mu &= \frac{1}{k} \text{tr} \left\{ -i(F^{\mu\nu} + \tilde{F}^{\mu\nu}) \xi \sigma_\nu \bar{\lambda} + \sqrt{2} D_\nu \bar{A} \xi \sigma^\nu \bar{\sigma}^\mu \psi + g \xi \sigma^\mu \bar{\lambda} [A, \bar{A}] \right. \\ &\quad \left. + (\xi \sigma_\nu \bar{\lambda}) C^{\mu\nu} \bar{\lambda} \bar{\lambda} - (\xi \sigma_\nu \bar{\lambda}) C^{\mu\nu} \{ \bar{A}, F \} \right\}.\end{aligned}\quad (9)$$

The supercurrent N_2^μ is given by

$$\begin{aligned}\eta N_2^\mu &= \frac{1}{k} \text{tr} \left\{ i(F^{\mu\nu} + \tilde{F}^{\mu\nu}) \eta \sigma_\nu \bar{\psi} + \sqrt{2} D_\nu \bar{A} \eta \sigma^\nu \bar{\sigma}^\mu \lambda - g \eta \sigma^\mu \bar{\psi} [A, \bar{A}] \right. \\ &\quad \left. - \frac{\sqrt{2}}{2} C^{\alpha\beta} \{ F^{\mu\nu} + \tilde{F}^{\mu\nu}, \bar{A} \} \eta_\alpha (\sigma_\nu \bar{\lambda})_\beta - C^{\mu\nu} \eta \sigma_\nu \bar{\lambda} (\bar{\lambda} \bar{\lambda} - \{ \bar{A}, F \}) \right. \\ &\quad \left. + i C^{\alpha\beta} \{ \bar{A}, D_\nu \bar{A} \} \eta_\alpha (\sigma^\nu \bar{\sigma}^\mu \psi)_\beta + i g \frac{\sqrt{2}}{2} C^{\mu\nu} \eta \sigma_\nu \bar{\lambda} \{ \bar{A}, [\bar{A}, A] \} \right. \\ &\quad \left. - i \frac{\sqrt{2}}{2} \det C \eta \sigma^\mu \bar{\lambda} (\{ \bar{A}, \bar{\lambda} \bar{\lambda} \} - \{ \bar{A}, \{ \bar{A}, F \} \}) \right\},\end{aligned}\quad (10)$$

which contains $O(C^2)$ corrections. For $C = 0$, we recover the undeformed supercurrents [19, 20]. The supercharge $Q_{i\alpha}$ is defined by

$$Q_{i\alpha} = \int d^3x N_{i\alpha}(x).$$

We now examine the anticommutation relations for supercharges $Q_{i\alpha}$. We will use the equal-time anticommutation relations for fermions

$$\{ \psi_\alpha(x), \bar{\psi}_{\dot{\alpha}}(y) \} = \delta_{\alpha\dot{\alpha}} \delta^3(x - y), \quad \{ \lambda_\alpha(x), \bar{\lambda}_{\dot{\alpha}}(y) \} = \delta_{\alpha\dot{\alpha}} \delta^3(x - y). \quad (11)$$

From (9), (10) and (11), we find that $\{Q_{1\alpha}, Q_{1\beta}\}$ and $\{Q_{1\alpha}, Q_{2\beta}\}$ are undeformed:

$$\{Q_{1\alpha}, Q_{1\beta}\} = 0, \quad (12)$$

$$\{Q_{1\alpha}, Q_{2\beta}\} = 2\sqrt{2}i\varepsilon_{\alpha\beta} \int d^3x \frac{1}{k} \text{tr} [(F_{0\ell} + \tilde{F}_{0\ell}) D^\ell \bar{A}]. \quad (13)$$

The r.h.s. of (13) comes from the 1st and 2nd terms in (9) and (10) and we have eliminated auxiliary fields by using the equations of motion. Eq. (13) is nothing but the central charge obtained by Witten and Olive [19].

The C -deformation arises in the anticommutation relation $\{Q_{2\alpha}, Q_{2\beta}\}$, which is given by

$$\{Q_{2\alpha}, Q_{2\beta}\} = 4C_{\alpha\beta} \int d^3x \frac{1}{k} \text{tr} [(F_{0\ell} + \tilde{F}_{0\ell}) D^\ell \bar{A}^2]. \quad (14)$$

The r.h.s. of (14) is obtained from the anticommutation relation among the 1st, 2nd, 4th and 7th terms in the current (10). Eq. (14) gives still the topological charge but its dependence on the vacuum expectation value of the Higgs fields is different from the undeformed topological charge (13).

In this paper, we will further study the deformed $\mathcal{N} = (1, 0)$ supersymmetry algebra for generic deformation case. In order to study deformed theories in extended superspace, it is convenient to introduce non(anti)commutative $\mathcal{N} = 2$ harmonic superspace [24] with supercoordinates $(x^\mu, \theta_i^\alpha, \bar{\theta}^{\dot{\alpha}i}, u^{\pm i})$. Here $i = 1, 2$ are $SU(2)_R$ R -symmetry group indices. $u^{\pm i}$ are the harmonic variables. The non(anti)commutativity in $\mathcal{N} = 2$ harmonic superspace [12] is introduced by

$$\begin{aligned} [x_L^\mu, x_L^\nu]_* &= [x_L^\mu, \theta_i^\alpha]_* = [x_L^\mu, \bar{\theta}^{\dot{\alpha}i}]_* = 0, \\ \{\bar{\theta}^{\dot{\alpha}i}, \bar{\theta}^{\dot{\beta}j}\}_* &= \{\bar{\theta}^{\dot{\alpha}i}, \theta_j^\alpha\}_* = 0, \quad \{\theta_i^\alpha, \theta_j^\beta\}_* = C_{ij}^{\alpha\beta}. \end{aligned} \quad (15)$$

where $C_{ij}^{\alpha\beta}$ is the deformation parameter. $x_L^\mu \equiv x^\mu + i\theta_i\sigma^\mu\bar{\theta}^i$ is the $\mathcal{N} = 2$ chiral coordinates. The non(anti)commutativity (15) is realized by using the $*$ -product:

$$f * g(\theta) = f(\theta) \exp\left(-\frac{1}{2} \overleftarrow{Q}_\alpha^i C_{ij}^{\alpha\beta} \overrightarrow{Q}_\beta^j\right) g(\theta). \quad (16)$$

where Q_α^i are supersymmetry generators which act on the $\mathcal{N} = 2$ superspace. The deformation parameter $C_{ij}^{\alpha\beta}$ has a symmetric property $C_{ij}^{\alpha\beta} = C_{ji}^{\beta\alpha}$ and can be decomposed as

$$C_{ij}^{\alpha\beta} = C_{(ij)}^{\alpha\beta} + \frac{1}{4} \epsilon_{ij} \epsilon^{\alpha\beta} C_s. \quad (17)$$

Here C_s corresponds to the singlet deformation and $A_{(ij)}$ denotes the symmetrized sum of A_{ij} over indices i and j .

At present the full component action for $\mathcal{N} = 2$ $U(N)$ gauge theory is not yet constructed for generic deformation parameters except for the singlet deformation case[22]. Recently, the exact form of the bosonic action of $U(1)$ theory is computed in [25] for the particular type of the non-singlet deformation $C_{(ij)}^{\alpha\beta} = c^{\alpha\beta} b_{(ij)}$. In the present work we will construct the supercharges for the $O(C)$ action of $\mathcal{N} = 2$ $U(1)$ gauge theory [13]. Although the supersymmetry algebra does not have central extension due to the absence of scalar potential, we would expect that the similar algebraic structure also appear in the $U(N)$ case.

The $O(C)$ Wess-Zumino gauge Lagrangian of the $U(1)$ theory [13] in the deformed $\mathcal{N} = 2$ harmonic superspace is given by

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{4}(1 + \sqrt{2}C_s\bar{\phi})F_{\mu\nu}(F^{\mu\nu} + \tilde{F}^{\mu\nu}) \\
& -i(1 - \frac{1}{\sqrt{2}}C_s\bar{\phi})\psi^i\sigma^\mu\partial_\mu\bar{\psi}_i + \frac{i}{\sqrt{2}}C_s\partial_\mu\bar{\phi}(\psi^i\sigma^\mu\bar{\psi}_i) \\
& +\phi\partial^2\bar{\phi} + \frac{1}{4}(1 - \sqrt{2}C_s\bar{\phi})D^{ij}D_{ij} \\
& -2\sqrt{2}iC_{(ij)}^{\alpha\beta}\psi_\alpha^i(\sigma^\mu\bar{\psi}^j)_\beta\partial_\mu\bar{\phi} - \frac{2\sqrt{2}}{3}iC_{(ij)}^{\alpha\beta}\psi_\alpha^i(\sigma^\mu\partial_\mu\bar{\psi}^j)_\beta\bar{\phi} \\
& +\frac{i}{2}C_s\bar{\psi}^i\bar{\psi}^jD_{ij} - iC_{(ij)}^{\mu\nu}\bar{\psi}^i\bar{\psi}^jF_{\mu\nu} + \frac{1}{\sqrt{2}}C_{(ij)}^{\mu\nu}D^{ij}F_{\mu\nu}\bar{\phi} + O(C^2). \tag{18}
\end{aligned}$$

We can compute the $O(C)$ contributions of $C_{(ij)}^{\alpha\beta}$ and C_s to the deformed supersymmetry separately. We firstly consider the case of non-singlet deformation $C_s = 0$. Setting $C_s = 0$, the Lagrangian (18) becomes

$$\begin{aligned}
\mathcal{L}^{non-singlet} = & -\frac{1}{4}F_{\mu\nu}(F^{\mu\nu} + \tilde{F}^{\mu\nu}) + \phi\partial^2\bar{\phi} - i\psi^i\sigma^\mu\partial_\mu\bar{\psi}_i + \frac{1}{4}D^{ij}D_{ij} \\
& -2\sqrt{2}iC_{(ij)}^{\alpha\beta}\psi_\alpha^i(\sigma^\mu\bar{\psi}^j)_\beta\partial_\mu\bar{\phi} - \frac{2\sqrt{2}}{3}iC_{(ij)}^{\alpha\beta}\psi_\alpha^i(\sigma^\mu\partial_\mu\bar{\psi}^j)_\beta\bar{\phi} \\
& -iC_{(ij)}^{\mu\nu}\bar{\psi}^i\bar{\psi}^jF_{\mu\nu} + \frac{1}{\sqrt{2}}C_{(ij)}^{\mu\nu}D^{ij}F_{\mu\nu}\bar{\phi} + O(C^2). \tag{19}
\end{aligned}$$

The $\mathcal{N} = (1, 0)$ supersymmetry transformation law in the WZ gauge is given in [26] by

$$\begin{aligned}
\tilde{\delta}_\xi\phi &= -\sqrt{2}\xi^i\psi_i - \frac{8}{3}i(\xi^i\varepsilon C_{(ij)}\psi^j)\bar{\phi} + O(C^2), \\
\tilde{\delta}_\xi\bar{\phi} &= 0, \\
\tilde{\delta}_\xi A_\mu &= i\xi^i\sigma_\mu\bar{\psi}_i + 2\sqrt{2}i(\xi^i\varepsilon C_{(ij)}\sigma_\mu\psi^j)\bar{\phi} + O(C^2), \\
\tilde{\delta}_\xi\psi^{\alpha i} &= -(\xi^i\sigma^{\mu\nu})^\alpha F_{\mu\nu} - D^{ij}\xi_j^\alpha - i(\xi^i\sigma_{\mu\nu})^\alpha C_{(jk)}^{\mu\nu}\bar{\psi}^j\bar{\psi}^k + 2\sqrt{2}D^{(ij}(\xi^k)\varepsilon C_{(jk)})^\alpha\bar{\phi} \\
&\quad - \{2\sqrt{2}(\xi^j\varepsilon C_{(jk)}\sigma^{\mu\nu})^\alpha + \frac{2\sqrt{2}}{3}(\xi^j\sigma^{\mu\nu}\varepsilon C_{(jk)})^\alpha + \sqrt{2}C_{(jk)}^{\mu\nu}\xi^{\alpha j}\}\epsilon^{ki}\bar{\phi}F_{\mu\nu} + O(C^2), \\
\tilde{\delta}_\xi\bar{\psi}_\alpha^i &= \sqrt{2}(\xi^i\sigma^\mu)_\alpha\partial_\mu\bar{\phi} + 2(\xi^j\varepsilon C_{(jk)}\sigma^\mu)_\alpha\partial_\mu\bar{\phi}^2\epsilon^{ki} + O(C^2), \\
\tilde{\delta}_\xi D^{ij} &= -2i\xi^i\sigma^\mu\partial_\mu\bar{\psi}^j - 6\sqrt{2}i\epsilon^{kl}\partial_\mu\{(\xi^i\varepsilon C_{(kl)}\sigma^\mu\bar{\psi}^j)\bar{\phi}\} \\
&\quad + 2\sqrt{2}i\epsilon^{il}\epsilon^{jm}(\xi^k\varepsilon C_{(lm)}\sigma^\mu\bar{\psi}_k)\partial_\mu\bar{\phi} + O(C^2). \tag{20}
\end{aligned}$$

Under the supersymmetric transformation (20), the Lagrangian (19) is invariant up to

$O(C)$. Then the supercurrent is

$$\begin{aligned}\xi^i \tilde{N}_i^\mu &= -i(F^{\mu\nu} + \tilde{F}^{\mu\nu})\xi^i \sigma_\nu \bar{\psi}_i - \sqrt{2}i(\psi'_i \sigma^\mu \bar{\sigma}^\nu \xi^i) \partial_\nu \bar{\phi} \\ &\quad - (2C_{(ij)}^{\mu\nu} \bar{\psi}^i \bar{\psi}^j + \sqrt{2}iC_{(ij)}^{\mu\nu} D^{ij} \bar{\phi}) \xi^k \sigma_\nu \bar{\psi}_k \\ &\quad - 2\sqrt{2}i(F^{\mu\nu} + \tilde{F}^{\mu\nu})\xi^i \varepsilon C_{(ij)} \sigma_\nu \bar{\psi}^j \bar{\phi} - 2i\xi^i \varepsilon C_{(ij)} \sigma^\nu \bar{\sigma}^\mu \psi'^j \partial_\nu \bar{\phi}^2 + O(C^2),\end{aligned}\quad (21)$$

where ψ' is defined by $\psi'_i{}^\alpha = \psi_i^\alpha - \frac{2\sqrt{2}}{3}C_{(ij)}^{\alpha\beta} \bar{\phi} \psi_\beta^j$. Using the anticommutation relation $\{\psi'_\alpha(x), \bar{\psi}_{j\beta}(y)\} = \delta_j^i \delta_{\alpha\beta} \delta^3(x-y)$, we find the deformation of the central charge as

$$\begin{aligned}\{Q_{i\alpha}, Q_{j\beta}\} &= 2\sqrt{2}\varepsilon_{ij}\varepsilon_{\alpha\beta} \int d^3x (F^{0\ell} + \tilde{F}^{0\ell}) \partial_\ell \bar{\phi} \\ &\quad + 8C_{(ij),\alpha\beta} \int d^3x (F^{0\ell} + \tilde{F}^{0\ell}) \partial_\ell \bar{\phi}^2 + O(C^2).\end{aligned}\quad (22)$$

The algebra (22) coincides with the $U(1)$ case of (12)–(14) under the reduction of the deformation parameter $C_{(ij)}^{\alpha\beta} = C_{11}^{\alpha\beta} \delta_i^1 \delta_j^1$.

We note that in the case of singlet deformation $C_{(ij)}^{\alpha\beta} = 0$, $C_s \neq 0$, the exact form of the Lagrangian is obtained in [22, 27] such as

$$\mathcal{L}^{singlet} = \left(1 + \frac{1}{\sqrt{2}}C_s \bar{\phi}\right)^2 \left[-\frac{1}{4}F_{\mu\nu}(F^{\mu\nu} + \tilde{F}^{\mu\nu}) + \phi \partial^2 \bar{\phi} - i\psi^i \sigma^\mu \partial_\mu \bar{\psi}_i + \frac{1}{4}D^{ij}D_{ij}\right].\quad (23)$$

Here the component fields in (23) except for A_μ and $\bar{\phi}$ are redefined from those in (18) so that the $\mathcal{N} = (1, 0)$ supersymmetry transformation law in the WZ gauge is the same as the undeformed one[22, 27]:

$$\begin{aligned}\hat{\delta}_\xi A_\mu &= i\xi^i \sigma_\mu \bar{\psi}_i, \quad \hat{\delta}_\xi \phi = -\sqrt{2}\xi^i \psi_i, \quad \hat{\delta}_\xi \bar{\phi} = 0, \\ \hat{\delta}_\xi \psi^{\alpha i} &= -(\xi^i \sigma^{\mu\nu})^\alpha F_{\mu\nu} - D^{ij} \xi_j^\alpha, \quad \hat{\delta}_\xi \bar{\psi}_\alpha^i = \sqrt{2}(\xi^i \sigma^\mu)_{\dot{\alpha}} \partial_\mu \bar{\phi}, \\ \hat{\delta}_\xi D^{ij} &= -i(\xi^i \sigma^\mu \partial_\mu \bar{\psi}^j + \xi^j \sigma^\mu \partial_\mu \bar{\psi}^i).\end{aligned}\quad (24)$$

The supercurrent generating the transformation (24) is given by

$$\xi^i \hat{N}_i^\mu = \left(1 + \frac{1}{\sqrt{2}}C_s \bar{\phi}\right)^2 \left[-i(F^{\mu\nu} + \tilde{F}^{\mu\nu})\xi^i \sigma_\nu \bar{\psi}_i - \sqrt{2}i(\psi_i \sigma^\mu \bar{\sigma}^\nu \xi^i) \partial_\nu \bar{\phi}\right].\quad (25)$$

From (25), we get the supersymmetry algebra as

$$\{Q_{i\alpha}, Q_{j\beta}\} = 2\sqrt{2}\varepsilon_{ij}\varepsilon_{\alpha\beta} \int d^3x \left(1 + \frac{1}{\sqrt{2}}C_s \bar{\phi}\right)^2 (F^{0\ell} + \tilde{F}^{0\ell}) \partial_\ell \bar{\phi}.\quad (26)$$

Since A_μ and $\bar{\phi}$ are not redefined, we can directly combine (22) and (26) into the central charge formula. Then we finally obtain the supersymmetry algebra in generic deformation case:

$$\begin{aligned}\{Q_{i\alpha}, Q_{j\beta}\} &= 2\sqrt{2}\epsilon_{ij}\epsilon_{\alpha\beta}\int d^3x (F^{0\ell} + \tilde{F}^{0\ell})\partial_\ell\bar{\phi} \\ &\quad + 8C_{ij,\alpha\beta}\int d^3x (F^{0\ell} + \tilde{F}^{0\ell})\partial_\ell\bar{\phi}^2 + O(C^2).\end{aligned}\quad (27)$$

The $O(C^2)$ term of r.h.s. in (27) is generally nonzero as we have seen in the case of the singlet deformation (26).

We also examine the the supersymmetry algebra in $\mathcal{N} = 2$ supersymmetric $U(N)$ gauge theory with singlet deformation. The Lagrangian $\mathcal{L}_{U(N)}^{singlet}$ obtained in [22] is of the form

$$\begin{aligned}\mathcal{L}_{U(N)}^{singlet} &= \frac{1}{k}\text{tr}\left[-\frac{1}{4}(L^2 F_{\mu\nu})F^{\mu\nu} - \frac{1}{4}(L^2 F_{\mu\nu})\tilde{F}^{\mu\nu} - (L^2\psi^i)\sigma^\mu D_\mu\bar{\psi}_i + (L^2\phi)D^2\bar{\phi}\right. \\ &\quad \left. + \frac{1}{4}(L^2 D_{ij})D^{ij} - \frac{g}{\sqrt{2}}(L^2\psi^i)[\bar{\phi}, \psi_i] + \frac{g}{\sqrt{2}}\bar{\psi}^i[L^2\phi, \bar{\psi}_i] - \frac{g^2}{2}(L[\phi, \bar{\phi}])^2\right] \\ &\quad + (\text{higher-derivative terms}),\end{aligned}\quad (28)$$

where the operator L is defined by

$$L = 1 + \frac{C_s}{2\sqrt{2}}\{\bar{\phi}, \cdot\}.\quad (29)$$

The higher-derivative terms in the Lagrangian (28) can be absorbed by suitable field redefinitions[22]. After these field redefinitions, as in the $U(1)$ case, the Lagrangian (28) is invariant up to the total derivative under the undeformed $\mathcal{N} = (1, 0)$ supersymmetry transformations:

$$\begin{aligned}\delta_\xi A_\mu &= i\xi^i\sigma_\mu\bar{\psi}_i, \quad \delta_\xi\phi = -\sqrt{2}\xi^i\psi_i, \quad \delta_\xi\bar{\phi} = 0, \\ \delta_\xi\psi_i &= \sigma^{\mu\nu}\xi_i F_{\mu\nu} + D_{ij}\xi^j - ig\xi_i[\phi, \bar{\phi}], \quad \delta_\xi\bar{\psi}_i = -\sqrt{2}\bar{\sigma}^\mu\xi_i D_\mu\bar{\phi}, \\ \delta_\xi D_{ij} &= -2i\{\xi_i\sigma^\mu D_\mu\bar{\psi}_j + \sqrt{2}g[\xi_i\psi_j, \bar{\phi}]\}.\end{aligned}\quad (30)$$

The variation of the Lagrangian becomes

$$\delta_\xi\mathcal{L}_{U(N)}^{singlet} = \partial_\mu X^\mu, \quad X^\mu = -\frac{1}{k}\text{tr}\left(g\xi^i\sigma^\mu\bar{\psi}_i[L^2\phi, \bar{\phi}]\right).\quad (31)$$

Here we used the formula $L[\bar{\phi}, U] = [\bar{\phi}, LU]$ for an arbitrary field U . The supercurrent is given by

$$\begin{aligned} \xi^i N_i^\mu &= \frac{1}{k} \text{tr} \left[-i \{ L^2 (F^{\mu\nu} + \tilde{F}^{\mu\nu}) \} \xi^i \sigma_\nu \bar{\psi}_i \right. \\ &\quad \left. - \sqrt{2} i \{ (L^2 \psi_i) \sigma^\mu \bar{\sigma}^\nu \xi^i \} D_\nu \bar{\phi} + g \xi^i \sigma^\mu \bar{\psi}_i [L^2 \phi, \bar{\phi}] \right]. \end{aligned} \quad (32)$$

We then obtain the deformation of the central charge as

$$\{Q_{i\alpha}, Q_{j\beta}\} = 2\sqrt{2} \epsilon_{ij} \epsilon_{\alpha\beta} \int d^3x \frac{1}{k} \text{tr} \left[\{ L^2 (F^{0\ell} + \tilde{F}^{0\ell}) \} D_\ell \bar{\phi} \right]. \quad (33)$$

The algebra (33) is reduced to (26) in the case of $U(1)$ gauge group.

In this paper we have studied the central extension of the deformed $\mathcal{N} = (1, 0)$ supersymmetry algebra. For $U(N)$ gauge group, we have obtained the C -deformed central charge in the cases of the deformed $\mathcal{N} = 1$ superspace and $\mathcal{N} = 2$ harmonic superspace with the singlet deformation. In generic deformation case, we have computed the $O(C)$ -correction to the central charge for the $U(1)$ gauge group. It is important to study the full action of the deformed $\mathcal{N} = 2$ theory in order to discuss the complete C -deformed central charges. It is an interesting problem to find monopole and dyon solutions and study how the BPS structure is modified by the non(anti)commutativity. It is also interesting to study physical effects of this non(anti)commutativity in the strong coupling region of the theory.

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